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Title: Solving discontinuous problems with pseudospectral methods

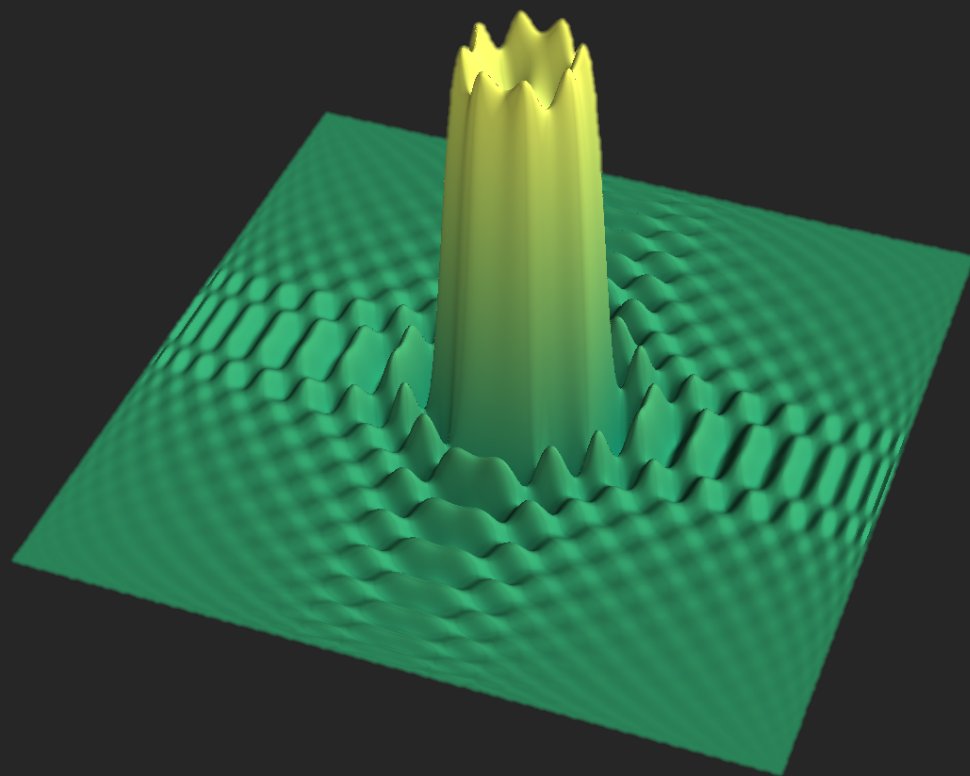
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Solving discontinuous problems with **pseudospectral methods**

JOANNA PIOTROWSKA, JONAH MILLER

Why investigate numerical methods?

... IF THERE ALREADY EXIST WIDELY APPLIED SCHEMES?

PSEUDOSPECTRAL METHODS

➤ Projection onto a set of basis functions:

$$(I_N f)(x) = \sum_{n=0}^N \phi_n T_n(x)$$

where:

$$\phi_n = \frac{1}{\gamma_n} \sum_{j=0}^N f(x_j) T_n(x_j) w_j,$$

$$\gamma_n = \sum_{j=0}^N T_n^2(x_j) w_j$$

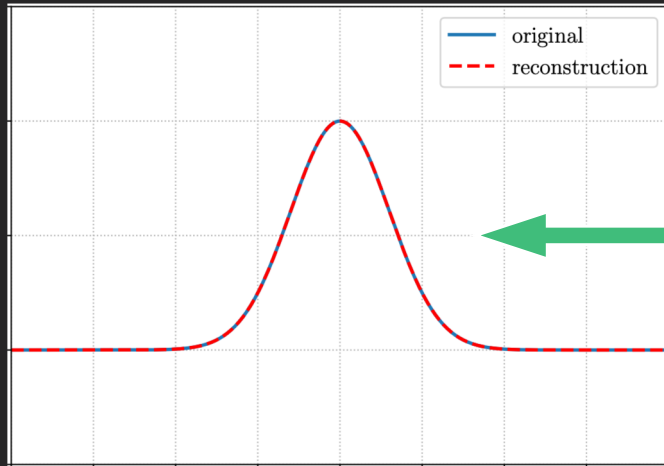
ADVANTAGES

Exponential error decay
for smooth solutions

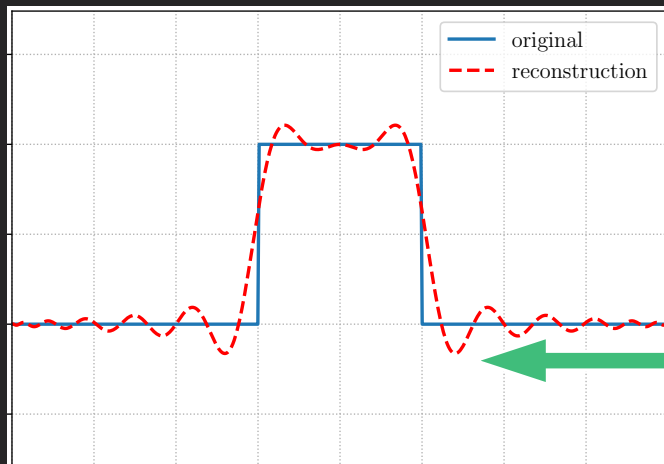
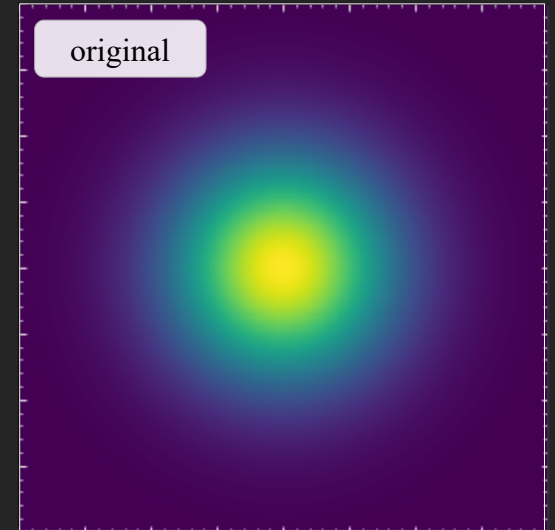
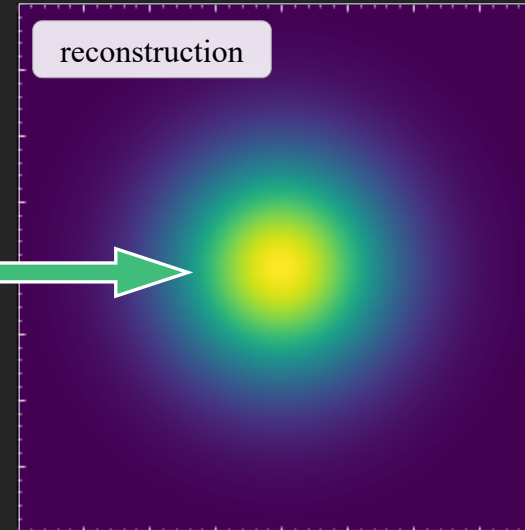
ISSUES

Gibbs phenomenon in
discontinuous solutions

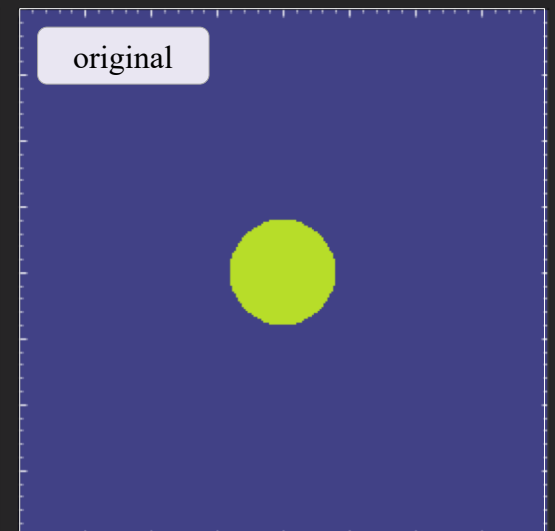
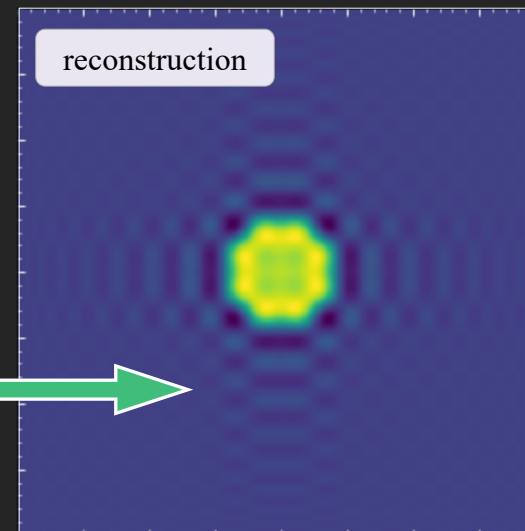
GIBBS PHENOMENON

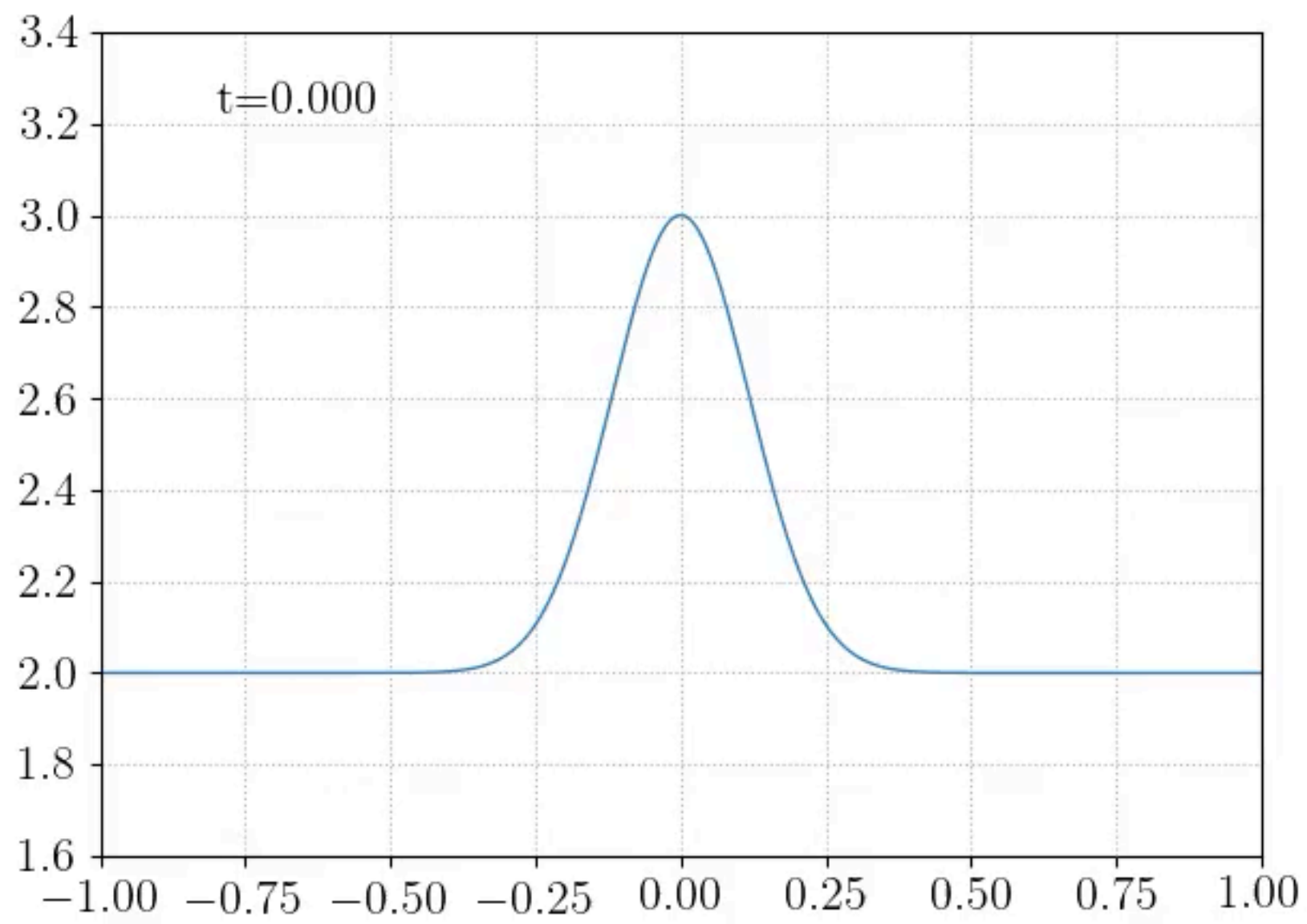


Smooth
reconstruction



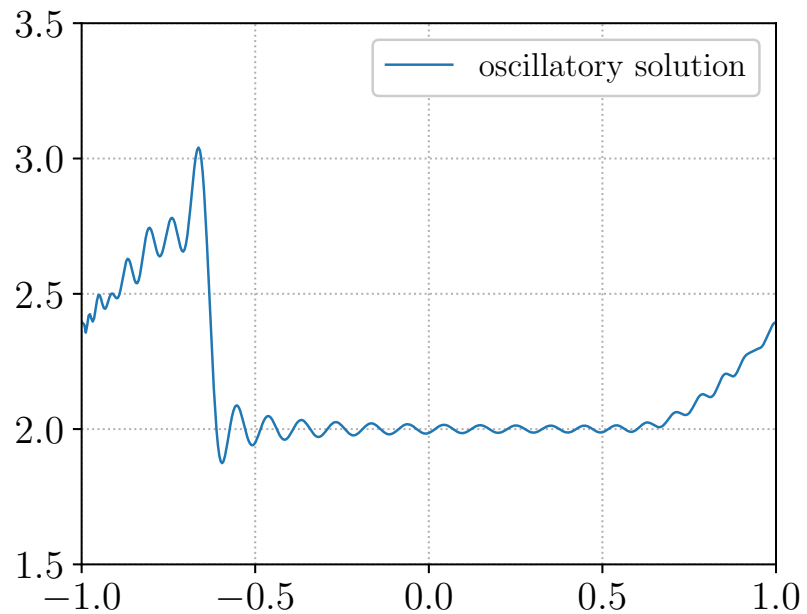
Gibbs
oscillations



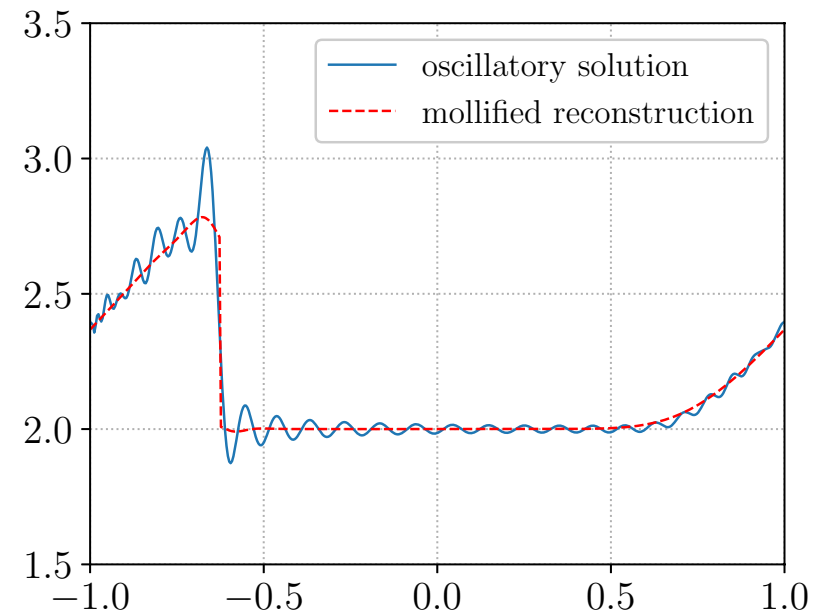


A system involving **shocks** can be solved with **pseudospectral methods**

Evolve the solution in time
at the collocation points

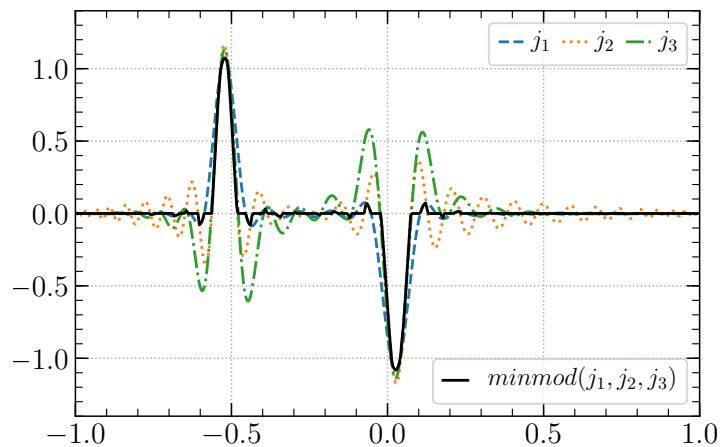
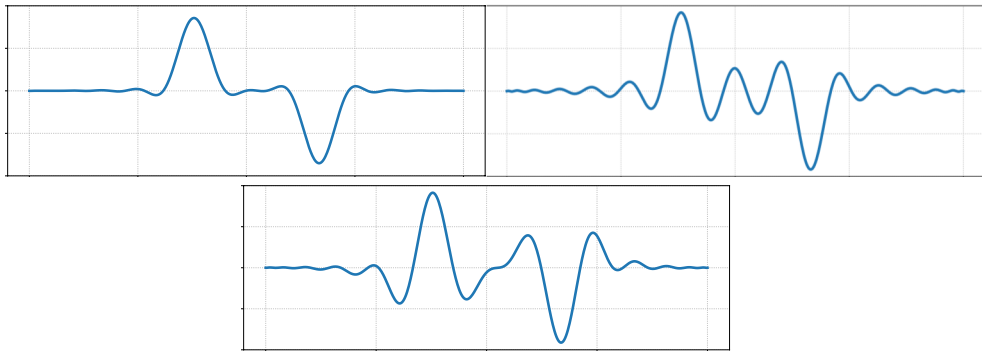


Remove Gibbs oscillations
in post-processing

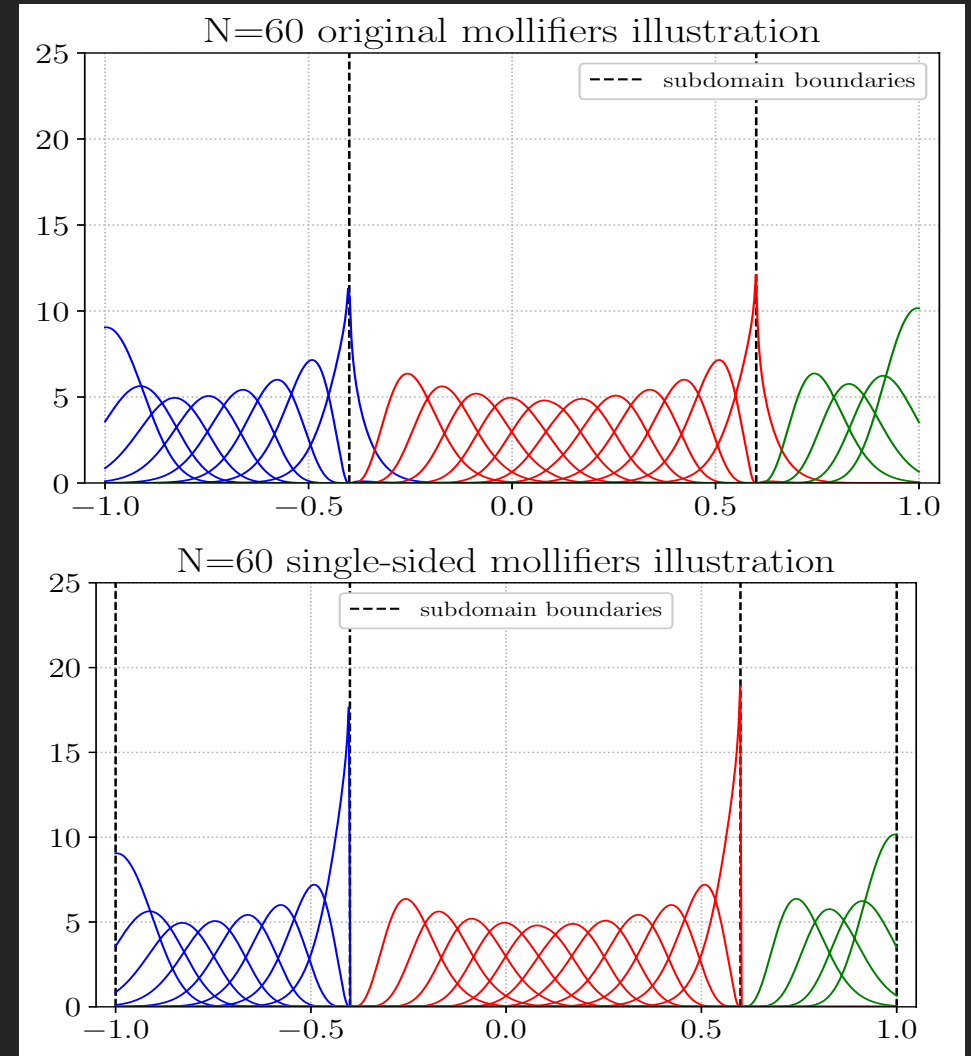


EDGE DETECTION

$$\frac{\sqrt{1-x^2}}{N} \sum_{k=1}^N \mu\left(\frac{k}{N}\right) \hat{f}(k) T'_k(x) \rightarrow [f](x)$$



MOLLIFICATION

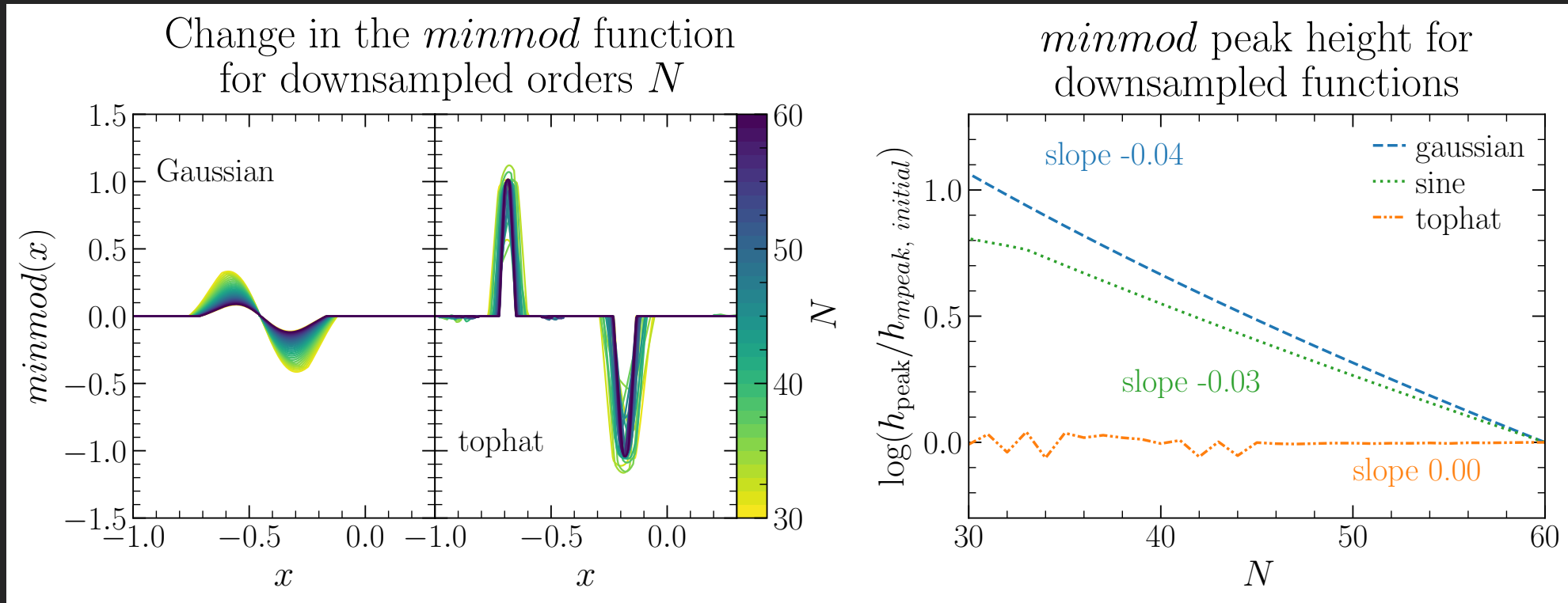


DYNAMICALLY EVOLVING DISCONTINUITIES require

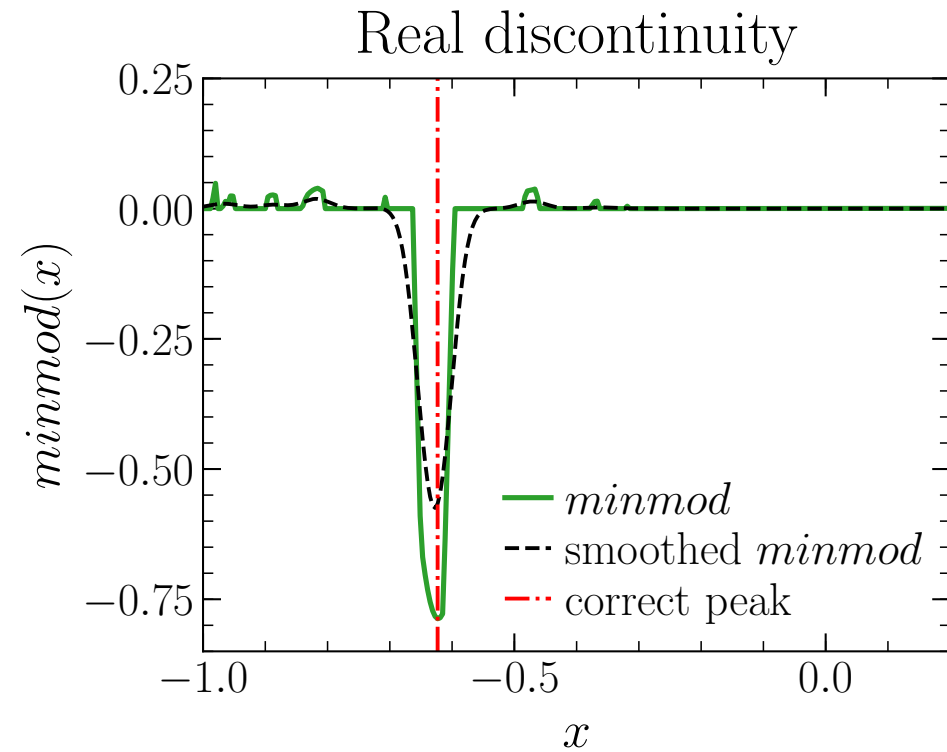
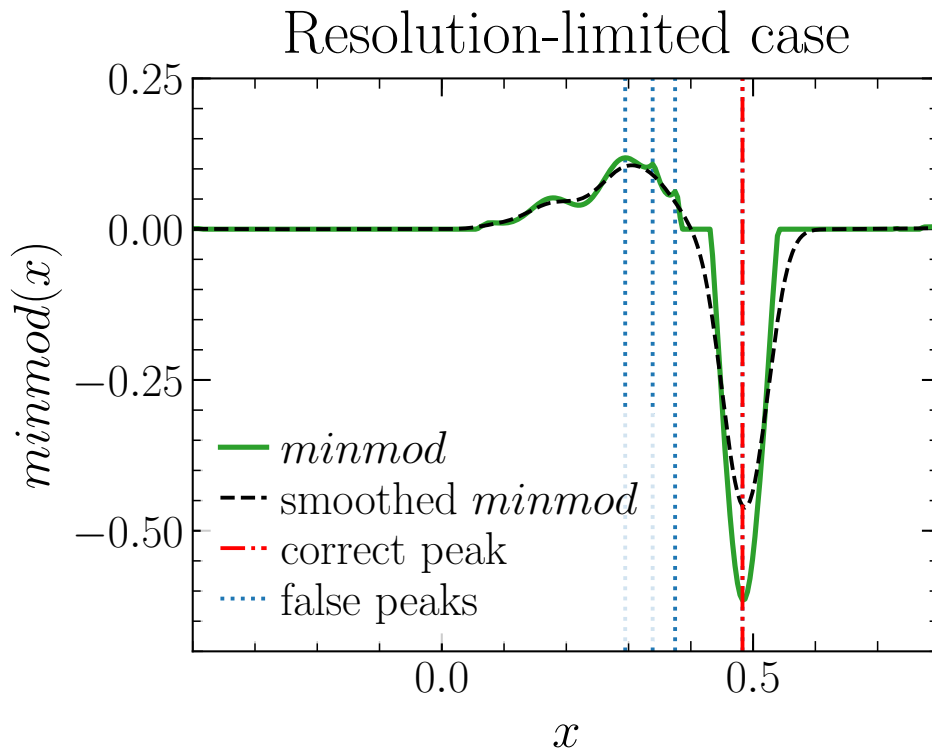
correct determination of
mollification onset

robust
edge detection

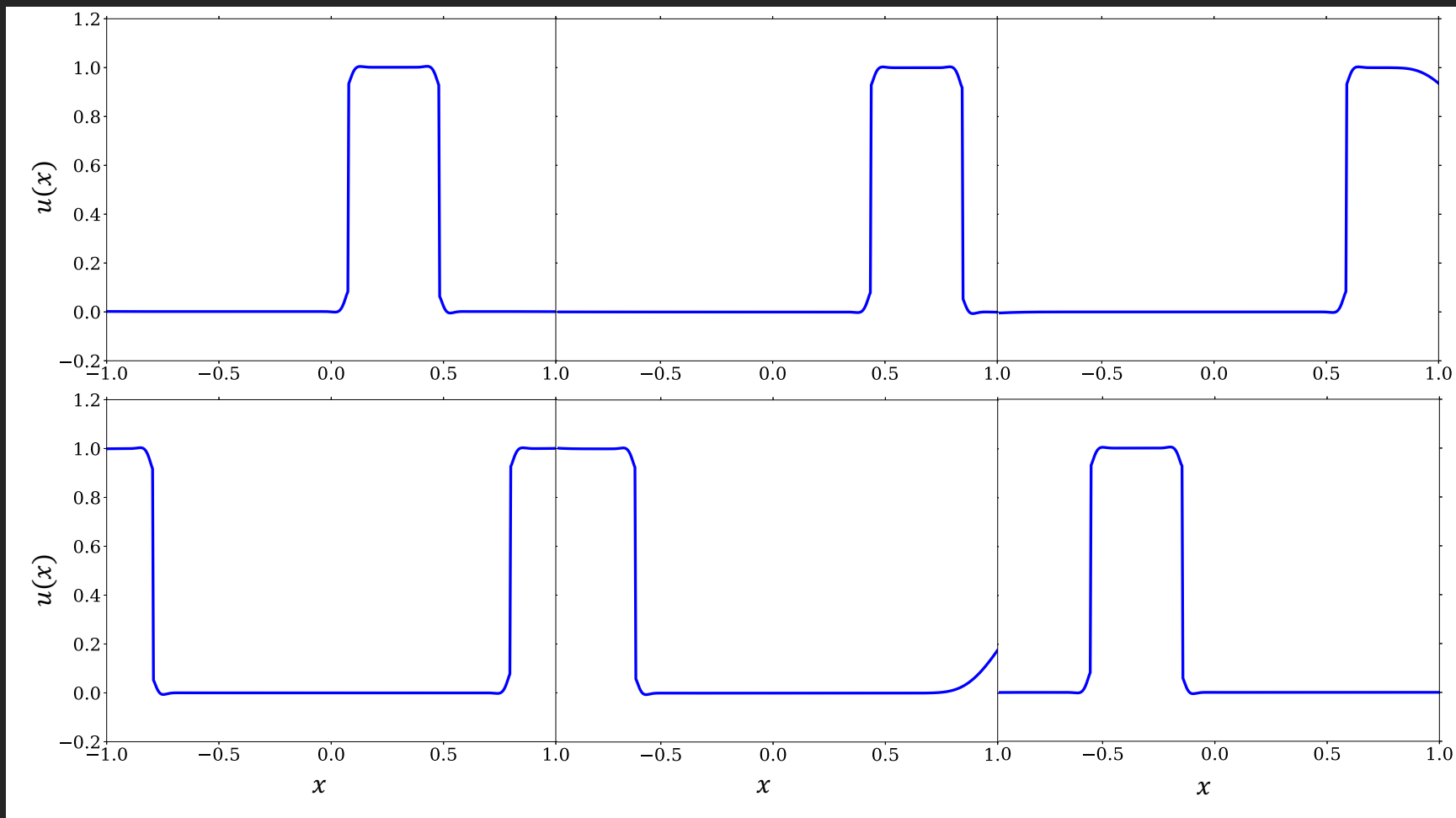
correct determination of **mollification onset**



robust edge detection

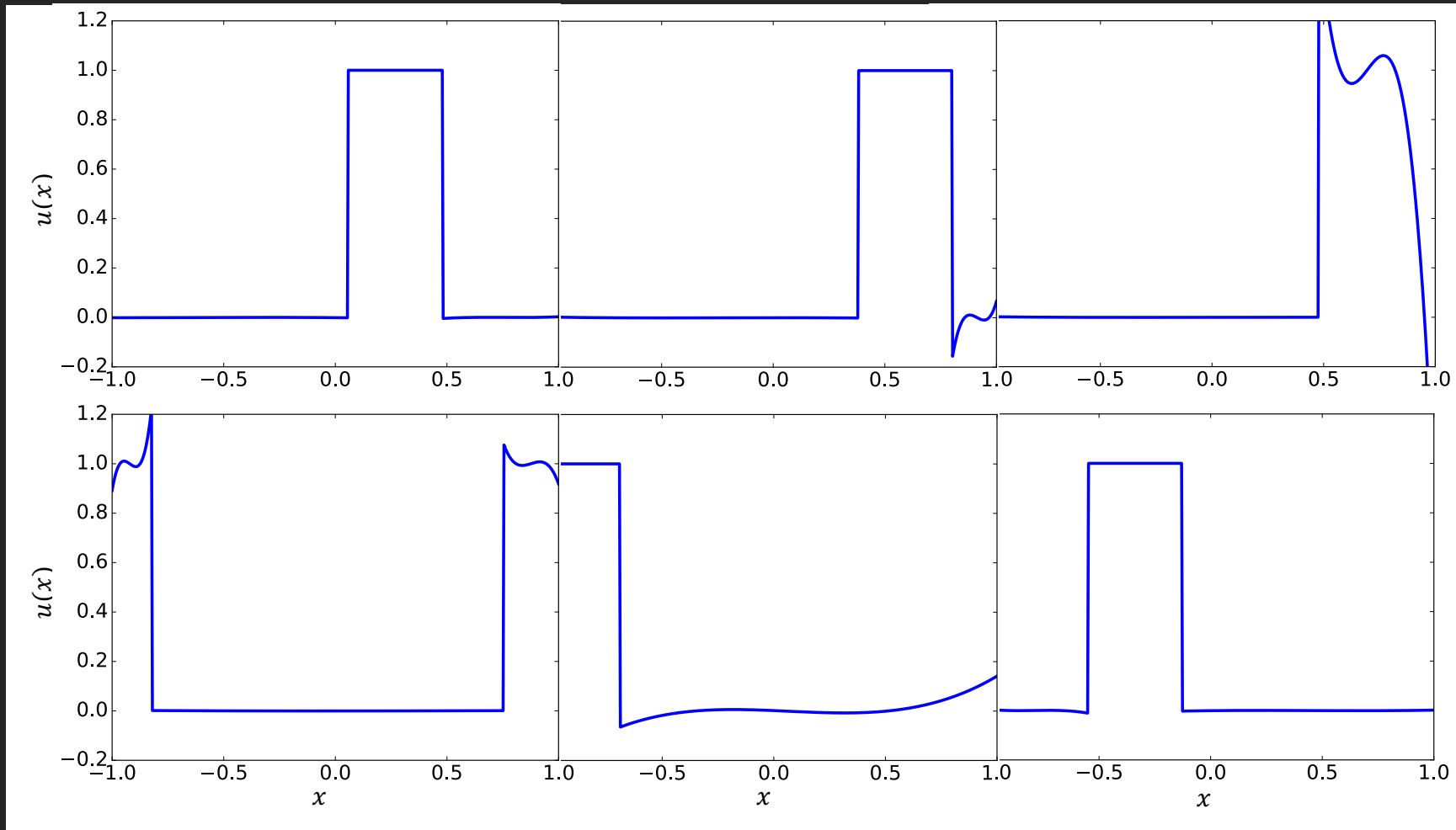


Toy example: 1D advection equation

$$\partial_t u + c \partial_x u = 0$$


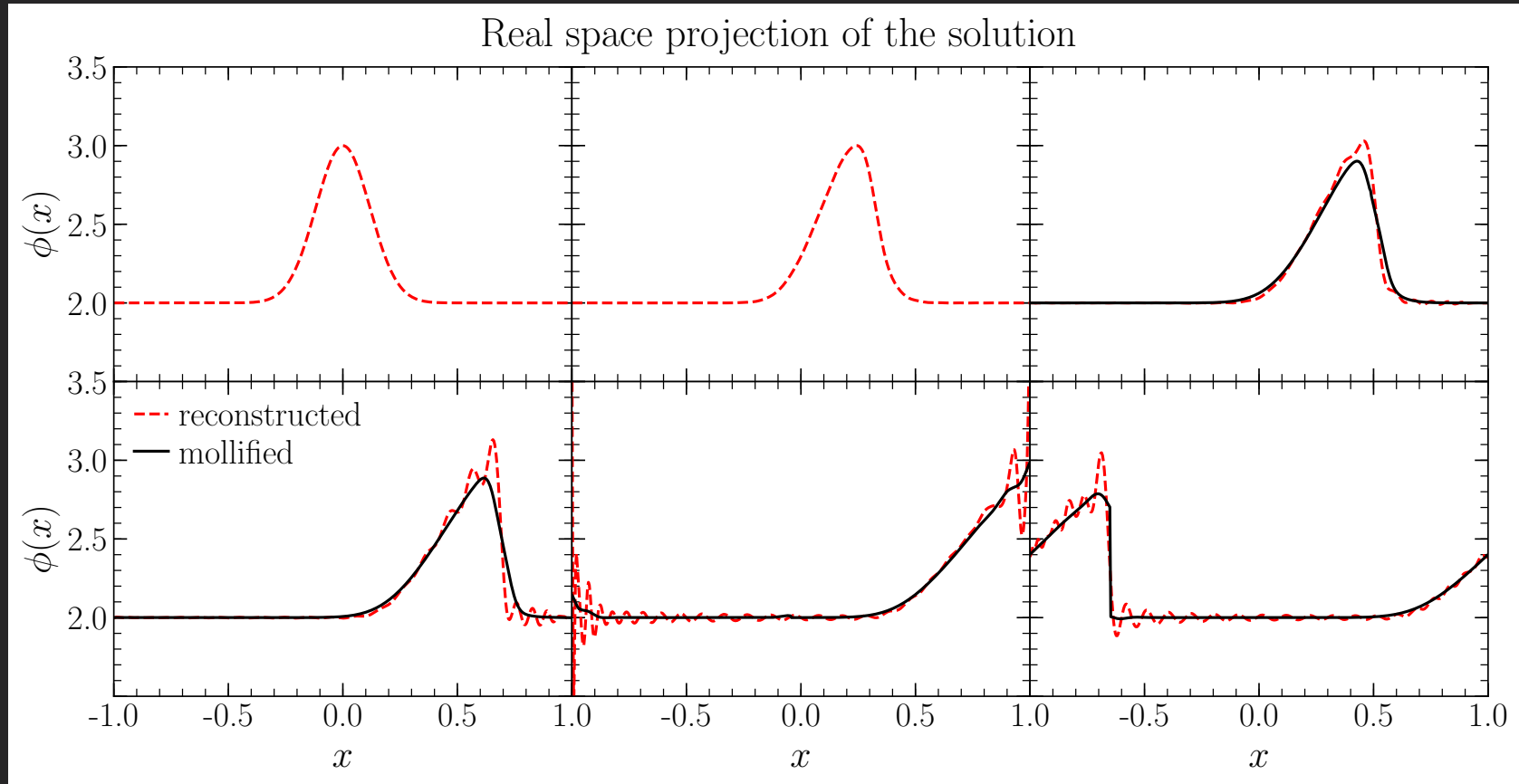
arXiv:1712.09952

comparison with the Gegenbauer reconstruction



courtesy of
J. M. Miller

Advanced toy example: 1D inviscid Burgers' equation:
$$\partial_t \phi + \phi \partial_x \phi = 0$$



SUMMARY & OUTLOOK

- **pseudospectral methods** are a **highly accurate** means of solving **continuous** problems
- they suffer the **Gibbs' phenomenon** in presence of **discontinuities**
- the Gibbs phenomenon can be **robustly removed** in post-processing via **edge detection** and **mollification** (arXiv:1712.09952, Piotrowska et al. in prep.)
- taking advantage of the robustness of mollification, we are currently **extending our working framework to 2D**

